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TURBULENT BOUNDARY LAYER OF POLYMER SOLUTIONS WITH  
A FLOW-RETARDING PRESSURE GRADIENT

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A description of the mean velocity distribution in the near-wall turbulent boundary layer of polymer solutions with a flow-retarding pressure gradient is given by using dimensional analysis and similarity.

Certain Peculiarities of Turbulent Boundary Layer Similarity

As is known [1] mean velocity distribution and drag law for turbulent flows is described with satisfactory accuracy by using dimensional analysis and similarity without detailing the turbulent model. The main reason for such simplicity is associated with the separation of the shear turbulent flow in a domain with substantially different scales, which are autonomous to a considerable degree and allows local description.

There are two characteristic length scales in the boundary layer on a smooth plate around which a viscous fluid flows with the constant velocity  $U$ : the boundary layer thickness  $\delta$  and the "viscous sublayer thickness" scale  $\delta_v \equiv \nu/u_*$  between which there exists the inequality  $\delta \gg \delta_v$  for a high degree of development of the turbulence. Using the lowest approximation for juncture of the asymptotic expansions in the small parameter  $\delta_v/\delta$ , a logarithmic formula for the mean velocity is derived successfully [1]. The derivation relies on the assumption of locality. It is considered that for a given  $x$  ( $x$  is the coordinate along the surface in the flow direction) the mean characteristics depend on  $\delta(x)$  and  $u_*(x)$  and are independent of their derivatives.

In the case of a boundary layer with a significant pressure gradient  $\alpha \equiv dp/dx$ , a third length scale  $\delta_\alpha \equiv u_*^2/\alpha$  becomes important (we neglect compressibility of the fluid and consider the density equal to one). It characterizes the length within which the friction stress on the streamlined surface  $u_*^2(x)$  varies substantially. Then the assumption of locality also implies slowness of the change in the velocity  $U(x)$ . Taking into account the steady flow equation  $\alpha(x) = -U(x)dU(x)/dx$ , the constraint on the derivative  $dU/dx$  can be given the form  $\alpha\delta \ll U^2$ .

Another important hypothesis is the assumption of autonomy of certain subdomains of the boundary layer. It results in the possibility of an independent analysis of the mean characteristics in these subdomains.

The autonomy condition actually reduces to similarity in the Reynolds number  $U\delta/\nu \gg 1$  for the outer flow zone; viscosity plays no part here. The assumption of autonomy in the inner subdomain (close to the surface being streamlined) denotes independence of the distribution from the external parameters  $U$ ,  $\delta$ ,  $\alpha$ . This can be expected when the inner and outer scales are not commensurate, at least the length ( $\delta_v \ll \delta$ ), the time ( $\nu/u_*^2 \ll \delta/U$ ), and the smallness of the change in the stress change  $u_*^2$  in the "viscous" length ( $\alpha\nu/u_* \ll u_*^2$ ). This latter assumption can be given the form of the condition  $\delta_v \ll \delta_\alpha$ .

For a sufficiently strong pressure gradient, its influence will penetrate deeply, and  $\delta_\alpha \ll \delta$ , as will also be taken into account later. If the time scales associated with the effect of viscosity, the pressure gradient, and the external flow  $\nu/u_*^2 \ll u_*/\alpha \ll \delta/U$  are also incommensurate, then the existence of an autonomous intermediate pressure subdomain can also be expected. Together with those considered earlier, these conditions result in the requirement  $u_*U \ll \alpha\delta \ll U^2$ , which imposes strong constraints on the possibility of an auto-

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mous description of the pressure subdomain. This is apparently why no complete similarity is detected in the measurements performed, and some refinement is required [2, 3]. Nevertheless, the locality and autonomy conditions are assumed satisfied in a further simplified consideration.

### Multiscale Boundary Layer Analysis of Polymer Solutions

Still another dimensional parameter characterizing the solution should be taken into account in describing the turbulent flows of solutions manifesting a drag reduction effect. We use the solution relaxation time  $\theta$  as such a parameter (the case of the length dimensionality parameter can be examined analogously [4]). Moreover, a certain dimensionless parameter  $\pi$ , for instance the polymer concentration, should be taken into account.†

The length scale  $\delta_\theta \equiv \theta u_*$  characterizing the size of the domain of inertial-elastic influence of the polymer on the turbulence can exceed the viscous  $\delta_\theta \gg \delta_\nu$  if the effect is strong. The scale  $(\nu\theta)^{1/2}$  characterizing the size of the viscous-elastic influence of the polymer is less substantial since it depends on the viscosity and has an intermediate value ( $\delta_\nu \ll (\nu\theta)^{1/2} \ll \delta_\theta$ ).

In conformity with the assumption on locality, the distribution of mean characteristics in a fixed boundary layer section  $x$  can depend on the governing parameters  $u_*(x)$ ,  $\delta(x)$ ,  $\alpha(x)$ ,  $\nu$ ,  $\theta$ ,  $\pi$ . It would seem that the quantity  $U(x)$  should also be included here as a characteristic of the external zone. However, the flow in this zone is independent of the viscosity, and in the ideal fluid approximation the absolute value of the velocity should not be substantial (see [1], also). Nevertheless,  $U$  can be in the final results when conditions on the outer boundary are taken into account.

If the influence of the pressure gradient does not occupy the inertial-elastic zone completely, i.e., the change in stress  $u_*^2$  can be neglected within its limits, then  $\delta_\alpha \gg \delta_\theta$ , and in conformity with the previous inequalities, we have the following relationship between the scales

$$\delta \gg \delta_\alpha \gg \delta_\theta \gg \delta_\nu. \quad (1)$$

Let us assume that such inequalities are satisfied and autonomous boundary-layer subdomains exist for whose description one of these characteristic length scales is sufficient. Let us examine these subdomains further.

The mean velocity gradient  $d \langle u \rangle / dy$  can depend on just the coordinate  $y$  and the dimensional parameters  $\delta$ ,  $\alpha$  for  $y \gg \delta_\alpha$  in the outer part of the boundary layer, so that

$$\frac{d \langle u \rangle}{dy} = \sqrt{\frac{\alpha}{y}} f_1 \left( \frac{y}{\delta} \right), \quad y \gg \delta_\alpha. \quad (2)$$

By using integration and application of the condition on the outer boundary ( $\langle u \rangle = U$  for  $y = \delta$ ), this formula can be given the form of a "velocity defect law"

$$\frac{U - \langle u \rangle}{v} = F \left( \frac{y}{\delta} \right), \quad y \gg \delta_\alpha. \quad (3)$$

For the case of a streamlining flow at a constant velocity,  $v = u_*$ , while for a boundary layer with a large pressure gradient  $v = \sqrt{\alpha\delta} \gg u_*$ .

In the next autonomous subdomain  $\delta \gg y \gg \delta_\theta$ , the statistical flow characteristics can depend on  $y$ ,  $\alpha$ ,  $\delta_\alpha$  (or  $y$ ,  $\alpha$ ,  $u_*$ ), and particularly

$$\frac{d \langle u \rangle}{dy} = \sqrt{\frac{\alpha}{y}} f_2 \left( \frac{y}{\delta_\alpha} \right), \quad \delta \gg y \gg \delta_\theta. \quad (4)$$

The properties of the polymer solution become important in the domain  $\delta_\alpha \gg y \gg \delta_\nu$  and mean velocity gradient can depend on  $y$ ,  $u_*$ ,  $\delta_\theta$ ,  $\pi$  (or  $y$ ,  $u_*$ ,  $\theta$ ,  $\pi$ ):

$$\frac{d \langle u \rangle}{dy} = \frac{u_*}{y} f_3 \left( \frac{y}{\delta_\theta}, \pi \right), \quad \delta_\alpha \gg y \gg \delta_\nu. \quad (5)$$

†The relaxation time can vary independently of the concentration changing the temperature of the solution.

Near a smooth surface the governing parameters for  $y \ll \delta_\theta$  will be  $y, u_*, \delta_\nu, \pi$  (or  $y, u_*, \nu, \pi$ ). Then

$$\frac{d\langle u \rangle}{dy} = \frac{u_*}{y} f_4 \left( \frac{y}{\delta_\nu}, \pi \right), \delta_\theta \gg y. \quad (6)$$

In these formulas the  $f_1, f_2, f_3, f_4, F$  are universal dimensionless functions of their dimensionless arguments.

Let us assume that three overlapping domains exist between the four zones mentioned  $\delta_\alpha \ll y \ll \delta, \delta_\theta \ll y \ll \delta_\alpha, \delta_\nu \ll y \ll \delta_\theta$ , in which the appropriate pairs of formulas are simultaneously valid. Then the form of the functions  $f_1, f_2, f_3, f_4$  is determined to the accuracy of coefficients independent of  $y$  in the overlap domains.

In the domain  $\delta_\alpha \ll y \ll \delta, f_1 = f_2 = K/2$  from the conjugate of the asymptotic dependences (2) and (4), and the velocity profile should have the form [2, 5]

$$\langle u^+ \rangle = K \sqrt{\frac{\alpha}{u_*^2} y} + M, \delta \gg y \gg \delta_\alpha. \quad (7)$$

Here  $K$  is a universal constant ( $K \approx 4.2$  according to [5]).

Analogously,  $f_3 (y/\delta, \pi) = \sqrt{\delta_\alpha/y} f_4 (y/\delta_\alpha) = A$  in the domain  $\delta_\theta \ll y \ll \delta_\alpha$ , and

$$\langle u^+ \rangle = A \ln y^+ + B, \delta_\alpha \gg y \gg \delta_\theta. \quad (8)$$

Here  $A \approx 2.5$  is a universal constant [1].

Finally, in the overlap domain  $\delta_\nu \ll y \ll \delta_\theta$ , it follows from the relationships (5) and (6) that  $f_3 = f_4 = A_2(\pi)$  and

$$\langle u^+ \rangle = A_2(\pi) \ln y^+ + B_2, \delta_\theta \gg y \gg \delta_\nu. \quad (9)$$

The velocity profile is also sufficiently universal in the nearest neighborhood of a smooth wall  $y \ll \delta_\nu$ . If we limit ourselves to the first term of the shear stress expansion in the velocity gradient, then  $\langle \tau \rangle = \nu d\langle u \rangle / dy$  just as for a viscous fluid, and therefore

$$\langle u^+ \rangle = y^+, \delta_\nu \gg y. \quad (10)$$

#### Approximate Description of the Velocity

Although the distributions (7)-(10) found are valid only in the appropriate subdomains, we use them for an approximate description of the mean velocity profiles in the whole boundary layer  $0 < y < \delta$ . Let us consider a continuous distribution comprised of the simple expressions (7)-(10) down to boundary points of the adjacent domains  $\Delta_\nu = c_1 \delta_\nu, \Delta_\theta = c_2 \delta_\theta, \Delta_\alpha = c_3 \delta_\alpha$ , and  $\delta$  ( $c_1, c_2, c_3$  are numerical factors, where  $c_1$  and  $c_2$  can generally depend on  $\pi$ ):

$$\langle u^+ \rangle = \begin{cases} y^+ & , 0 \leq y^+ \leq \Delta_\nu^+, \\ A_2(\pi) \ln y^+ + B_2 & , \Delta_\nu^+ \leq y^+ \leq \Delta_\theta^+, \\ A \ln y^+ + B & , \Delta_\theta^+ \leq y^+ \leq \Delta_\alpha^+, \\ K \sqrt{\frac{\alpha \nu}{u_*^2} y^+} + M & , \Delta_\alpha^+ \leq y^+ \leq \delta^+ . \end{cases} \quad (11)$$

The greatest deviations of the real data from this model profile (shown schematically in the figure) can exist near the junctures. However, as a rule, the accuracy of predictions for low-concentration polymer solutions near the point  $\Delta_\nu^+$  is not less than for the usual viscous fluid. The exact position of the point  $\Delta_\alpha^+$  is barely essential because of the closeness of the logarithmic and root distributions. The error related to continuation of the root distribution down to the outer boundary  $\delta$  can be diminished by the introduction of an external sublayer (see below).

The conditions  $B_2 = c_1 - A_2 \ln c_1, B = (A_2 - A) \ln \left( c_2 \theta \frac{u_*^2}{\nu} \right) + B_2,$

$$M = -K \sqrt{c_3} + A \ln \left( c_3 \frac{u_*^3}{\alpha \nu} \right) + B, U^+ = M + K \sqrt{\frac{\alpha \delta}{u_*^2}} \quad (12)$$

should be satisfied for continuity of the mean velocity profiles at the points  $\Delta_\nu, \Delta_\theta, \Delta_\alpha, \delta$ .

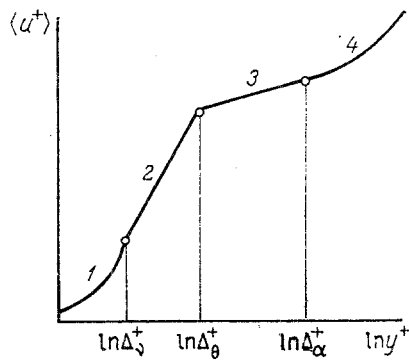


Fig. 1. Model representation of the mean velocity profile: 1)  $\langle u^+ \rangle = y^+$ ; 2)  $\langle u^+ \rangle = A_2 \ln y^+ + B_2$ ; 3)  $\langle u^+ \rangle = A \ln y^+ + B$ ; 4)  $\langle u^+ \rangle = K \sqrt{y^+ \alpha \nu / u_*^3} + M$ .

Taking account of the last relationship, the velocity profile can be written in the form of (3) in the outer boundary layer zone, where

$$F\left(\frac{y}{\delta}\right) = K\left(1 - \sqrt{\frac{y}{\delta}}\right), \quad y > \Delta_\alpha. \quad (13)$$

From (12), the logarithmic formula for B is simplified if the natural conditions for making the transition to the case of a viscous fluid are taken. Let us assume that as  $\Delta_\theta^+$   $\Delta_\nu$  the distribution (11) goes over into the appropriate distribution for a viscous fluid with  $B = B_0 \approx 5.5$ ,  $A \approx 2.5$ . Then

$$c_1 \approx 11.6; \quad A_2 \approx 2.5 + 0.5a(\pi), \quad B_2 \approx 5.5 - 1.23a(\pi), \quad (14)$$

$$B \approx 5.5 + \begin{cases} 0, & u_* < u_{*cr}, \\ a(\pi) \ln \frac{u_*}{u_{*cr}}, & u_* \geq u_{*cr}. \end{cases}$$

The quantity  $u_{*cr}^2 \equiv c_1 \nu / c_2 \theta$  has the meaning of a critical stress at which the substantial influence of the polymer on the mean velocity profile starts. In terms of it  $\Delta_\theta$  can be described:

$$\Delta_\theta^+ = 11.6 (u_* / u_{*cr})^2, \quad u_* \geq u_{*cr}. \quad (15)$$

Formula (14) for B was found empirically in [6]. The quadratic dependence of  $\Delta_\theta^+$  on  $u_*$  is seen from the experimental data in [7]. Let us note that this dependence would be linear [4] under an additional parameter of the dimensionality of a length.

The magnitude of the coefficient  $a(\pi)$  can vary significantly. However, the requirement of no points of inflection on the velocity profile, associated with the condition of flow stability, imposes the constraint  $a(\pi) \leq 18.2$ , which is equivalent to  $A_2 \leq 11.6$  or  $B_2 \geq -16.9$  [4].

As the pressure gradient  $\alpha$  diminishes, the logarithmic distribution gradually displaces the root distribution to the outer boundary, and we finally obtain  $\delta = \Delta_{\alpha cr} = c_3 u_*^2 / \alpha_{cr}$ ,  $U^+ = A \ln(\delta u_* / \nu) + B$  for some  $\alpha = \alpha_{cr}$ . Hence, for a minimum pressure gradient at which the substantial influence of the pressure on the boundary layer starts, we have the formula

$$\frac{\alpha_{cr} \nu}{u_*^3} = \frac{c_3}{\delta^+} = 9c_3 \exp(-0.4U^+) \begin{cases} 1, & u_* < u_{*cr}, \\ \left(\frac{u_*}{u_{*cr}}\right)^{0.4a(\pi)}, & u_* \geq u_{*cr}. \end{cases} \quad (16)$$

This criterion turns out to be responsive to the presence of a polymer.

The formulas for M and  $U^+$  can be rewritten as follows for  $\alpha \geq \alpha_{cr}$ ,  $u_* \geq u_{*cr}$  by using the parameter  $\alpha_{cr}$ :

$$\begin{aligned} M &= A \ln\left(\frac{\alpha_{cr}}{\alpha} \delta^+\right) - K \sqrt{\frac{\alpha_{cr} \nu}{u_*^3} \delta^+} + B, \\ U^+ &= A \ln\left(\frac{\alpha_{cr}}{\alpha} \delta^+\right) + K \sqrt{\frac{\alpha_{cr} \nu}{u_*^3} \delta^+} \left(\sqrt{\frac{\alpha}{\alpha_{cr}}} - 1\right) + B, \\ B &= a(\pi) \ln\left(\frac{u_*}{u_{*cr}}\right) + B_0. \end{aligned} \quad (17)$$

Hence, it is seen that within the framework of the model under consideration, the contribution to  $U^+$  from the influences of the polymer and the pressure gradient are additive. If  $u_*$  or  $\alpha$  turn out to be below the critical levels, then the influence of the polymer or the pressure gradient on the boundary layer vanishes and the corresponding members drop out of the formulas.

By introducing the local friction coefficient  $c_f = 2u_*^2/U^2$  and the Reynolds number  $Re = U\delta/\nu$ , the formula for  $U^+$  can be rewritten in the form of a "drag law" for  $A \approx 2.5$ ;  $B_0 \approx 5.5$ ;  $K \approx 4.2$ :

$$\frac{2}{\sqrt{c_f}} = 5.5 + 2.5 \ln \left( Re \sqrt{\frac{c_f}{2}} \right) + 4.2 \sqrt{\frac{2Re}{c_f}} \left( \sqrt{\frac{\alpha\nu}{U^3}} - \sqrt{\frac{\alpha_{cr}\nu}{U^3}} \right) + a(\pi) \ln \left( \frac{U}{u_{*cr}} \sqrt{\frac{c_f}{2}} \right) + 2.5 \ln \left( \frac{\alpha_{cr}}{\alpha} \right), \quad c_f \geq 2 \frac{u_{*cr}^2}{U^2}, \quad \alpha \geq \alpha_{cr}. \quad (18)$$

We obtain an estimate of the maximum influence of the polymer on the boundary layer from the condition that the thickness of the polymer zone  $\Delta_\theta$  becomes as great as possible, while the slope of the profile in this zone reaches its limit with  $\alpha(\pi) = 18.2$ . If it is assumed that the polymer exerts no influence on the pressure zone ( $y > \Delta_\alpha$ ), as on the "outer zone," then the limit condition will be when  $\Delta_\theta$  reaches  $\Delta_\alpha$  and the velocity profile corresponding to the "limit asymptote" conditions will be

$$\langle u^+ \rangle = \begin{cases} y^+, & 0 \leq y^+ \leq 11.6, \\ 11.6 \ln y^+ - 16.9, & 11.6 \leq y^+ \leq \Delta_\alpha^+, \\ K \sqrt{\frac{\alpha\nu}{u_*^3}} y^+ + M, & \Delta_\alpha^+ \leq y^+ \leq \delta^+. \end{cases} \quad (19)$$

The polymer generally ceases to influence the velocity profile when  $\Delta_\alpha^+$  reaches the value 11.6 for a significant pressure gradient, and therefore, does not influence the turbulent flow drag. This occurs for  $\alpha = \alpha_0 \equiv c_3 u_*^3 / 11.6\nu$ .

The requirement of no inflection points on the velocity profile at the point  $y = \Delta_\alpha$  results in the constraint  $K\sqrt{c_3} \leq 2A \approx 5$ , where the equality sign corresponds to the case of a smooth passage from the logarithmic to the root law. The experimental results indicate that almost this case is realized [2, 5].\* For  $K = 4.2$  we obtain the estimate  $c_3 \approx 1.4$  (compare with [5]). The error associated with continuation of the root distribution to the outer boundary layer boundary can be diminished by insertion of an outer sublayer. For an insignificant pressure gradient in a viscous fluid, the velocity profile in such a sublayer deviates from the logarithmic and is described well by a velocity defect law (3) with  $v = u_*$  and with the empirical function  $F(y/\delta) = 9.6 (1 - y/\delta)^2$ , if  $y > 0.158$  (see [1]).

The same distribution satisfactorily describes the outer sublayer even in the presence of a pressure gradient [2] (then  $v = \sqrt{\alpha\delta}$ ). In the subdomain  $\Delta_\alpha \leq y \leq 0.15\delta$  in which the "velocity defect law" is also valid,  $F(y/\delta) = M_1 - K\sqrt{y/\delta}$ , where  $M_1 \approx 8.6 \approx 2K$  because of the continuity of the velocity distribution. On the other hand, a representation in the form of (11) is allowable in the zone of influence of the pressure gradient. Compatibility of these two formulas is observed if  $U^+ = M + M_1\sqrt{\alpha\delta}/u_*^2$ . In the case under consideration such a relationship replaces the last formula in (12) that will result in only moderate changes in the numerical coefficient in the drag formula. It is also easy to trace other changes induced by taking account of an outer sublayer.

#### NOTATION

$U$ , external flow velocity;  $u_*$ , dynamic velocity;  $\nu$ , kinematic velocity coefficient;  $\delta$ , boundary layer thickness;  $\delta_\nu = \nu/u_*$ , viscous sublayer thickness scale;  $x$ , coordinate along the surface in the flow direction;  $\alpha = dp/dx$ , pressure gradient;  $p$ , pressure;  $\delta_\alpha = u_*^2/\alpha$ , a length scale characterizing the influence subdomain of the pressure gradient;  $\theta$ , relaxation time of the polymer solution;  $\delta_\theta = \theta u_*$ , length scale characterizing the subdomain of inertial-elastic influence of the polymer;  $\pi$ , a dimensionless parameter characterizing the polymer solution;  $y$ , coordinate normal to the surface;  $F, f_i$  ( $i = 1, 2, 3, 4$ ), dimensionless functions of dimensionless variables;  $\langle u \rangle$ , mean velocity;  $v$ , a parameter of the dimensionality of a velocity in the velocity defect law;  $Re$ , Reynolds number;  $u^+ = u/u_*$ ,  $y^+ =$

\*This is indeed conceivable from general considerations since pre-separation flow with a low stability margin is considered.

$y u_*^+ / \nu$ ,  $\delta^+ = \delta u_*^+ / \nu$ ,  $U^+ = U / u_*^+$ ,  $\theta^+ = \theta u_*^2 / \nu$ ,  $\Delta_V^+ = \Delta_V u_*^+ / \nu$ ,  $\Delta_\theta^+ = \Delta_\theta u_*^+ / \nu$ ,  $\Delta_\alpha^+ = \Delta_\alpha u_*^+ / \nu$ , dimensionless quantities in the near-wall variables; A, K, B<sub>0</sub>, universal constants in the logarithmic and root-mean velocity distributions; c<sub>1</sub>, c<sub>2</sub>, c<sub>3</sub>, numerical coefficients; u<sub>\*CR</sub>, stress at which the influence of the polymer on the mean velocity profile starts;  $\alpha(\pi)$ , parameter characterizing the influence of the polymer;  $\alpha_{CR}$ , minimal pressure gradient at which the influence of the pressure on the boundary layer starts; c<sub>f</sub>, local friction coefficient.

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#### HEAT EXCHANGE DURING FLOW OF ANOMALOUSLY VISCOUS FLUIDS IN CYLINDRICAL CHANNELS OF SIMPLY CONNECTED CROSS SECTION

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A method is proposed and results of a numerical solution are presented for a problem of heat exchange on the initial section of cylindrical channels of simply connected cross section during steady-state flow of an anomalously viscous fluid.

A theoretical investigation of heat exchange during flow of an anomalously viscous fluid in cylindrical channels of simply connected cross section has great applied importance.

A considerable number of studies [1, 2] have been devoted to questions of the heat exchange of anomalously viscous media for their flow in prismatic simply connected channels. However, in connection with the fact that the treatment of the given question encounters large mathematical difficulties, the known studies have either been of an experimental nature or have been devoted to a consideration of particular cases (a "power" rheological law, flow in channels of simplest forms, etc.). A fundamental obstacle for calculating the heat exchange in prismatic channels is the absence of analytical methods of determining the velocity profile in an anomalously viscous medium.

The aim of the present study is to solve the problem of heat exchange on the initial section of a cylindrical simply connected channel for flow of an anomalously viscous fluid described by an arbitrary rheological law for the case of boundary conditions of the first kind.

Considering laminar steady-state flow of an anomalously viscous fluid in a prismatic channel for the condition that heat transfer owing to heat conduction along the axis of the channel is incommensurably small in comparison with the forced transfer and dissipative release of heat is insignificant, the problem can be formulated in the following way:

$$v(x, y) \frac{\partial U}{\partial z} = a \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right), \quad (1)$$

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